Recreational Mathematics: A Short Introduction

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It is not easy to define these two innocent-looking words: *recreational mathematics*. As expressed by Charles Trigg (1978), "defining *recreational mathematics* is not recreational. The difficult task of defining *mathematics* is not simplified by the qualifying *recreational*." In this short note, I'm not even trying to define recreational mathematics. If we look through the lens of history, "behind the single expression of *mathematical recreations* one can identify, at different time periods, different enterprises inspired by different goals and developed by different types of actors" (Chemla, 2014). Therefore, let us try to understand what could be and what to expect from recreational mathematics.

Recreational mathematics cannot be understood as mathematics that one does for fun and amusement. Most professional mathematicians do enjoy their work and find mathematical research to be entirely pleasurable. Ian Stewart (2007) summarises, "People sometimes try to sell the idea that *mathematics can be fun*. I think that gets the emphasis wrong. To me, mathematics is fun." However, by any stretch of the imagination, it is impossible to include most of their subjects in the domain of recreational mathematics. Subjects such as Hecke algebra, holomorphic separability, harmonic morphism, and may more such involved concepts cannot possibly be called recreational.

The notion of recreational mathematics cannot be restricted within the domain of mathematical problems that require only elementary mathematical operations. Mere level of mathematical complexity involved does not describe the subject. Imagine that you walked into a supermarket, and the scanner machine is not working. You have to total costs of all the items and pay to the person at the counter. I'm sure totaling costs of these items will fail to meet anyone's concept of recreation, although it involves only simple mathematical procedures. On the other hand, several of the problems that lie in the domain of recreational mathematics can call for advanced and complicated mathematics for their solution, for example, Fermat's Last Theorem.

Around 250 AD, Diophantus wrote a book called *Arithmatica* that consists of several exercises in elementary number theory. In Book II, problem 8 challenges readers "to divide a given square number into two square numbers" (Hearth, 1964). For example, 25, square of 5, can expressed as sum of 16 (square of 4) and 9 (square of 3). This can be expressed in modern notations as $5^2 = 4^2 + 3^2$. Pierre de Fermat (1601-1665) had a Latin translation of *Arithmatica*, written by Bachet in 1621. He made many fascinating annotations in the margins of the book. While thinking about the sum of squares problem, sometimes in the late 1630s, he added the following famous words in the margin:

> *On the other hand, it is impossible to separate a cube into two cubes, or a biquadrate into two biquadrates, or generally any power except a square into two powers with the same exponent. I have discovered a truly marvellous proof of this, which however the margin is not large enough to contain.*

(This version of the statement is adopted from Barrow-Green et al., 2019). This particular hypothesis remained unsolved for nearly 350 years. Mathematicians could prove this theorem only for certain exponents. This problem became popular with the name Fermat's Last Theorem. Finally, in 1995, Prof. Andrew Wiles announced the first proof of this theorem. The argument delineated by Wiles is a clever blend of various advanced topics in number theory: elliptic curves, modular forms, and Galois representation.

Recreational mathematics mainly deals with those entertaining problems if they are easy to ask but difficult to answer. The problem statement must be in a form easily understood without advanced mathematical training. To understand the Fermat's Last Theorem, no formal training in mathematics is required. On the other hand, the problem should not have an immediately obvious solution. That is why totaling the costs of several items is not recreational. However, if the solution is too hard as well as calls for much-advanced mathematics, this may move the problem from recreational to serious. In this sense, Fermat's Last Theorem is no longer considered recreational.

Another overlapping notion of recreational mathematics, along with the previous one, covers much of the topics that are considered as recreational. This second notion emphasizes that recreational mathematics should be enjoyable where one is expected to put some cognitive efforts, linked with some positive motivations and emotions (Sumpter, 2015). Recreational mathematics can also be employed either as a departure from complicated and harder advanced stuff or as a means to make such difficult and tougher mathematics comprehensible and interesting. George Polya (1887-1985) opined that "a mathematics problem may be as much fun as a crossword puzzle, or that vigorous mental work may be an exercise as desirable as a fast game of tennis. Having tasted the pleasure in mathematics he will not forget it easily and then there is a good chance that mathematics will become something for him: a hobby, or a tool of his profession, or his profession, or a great ambition." Through these simple words, Polya highlighted the pedagogical efficacy of recreational mathematics. This was recognized by the ancient teachers and we could find them all over the oldest known mathematical works. This pedagogical efficacy is recognized even in the present day. Different school texts include various aspects of recreational mathematics to make learning mathematics fun. In India, a significant emphasis is put on the recreational aspects of mathematics to develop a positive attitude towards the subject (NCERT, 2005). Recreational mathematics also influenced many other related fields such as computer programming (Jiménez and Muñoz, 2011).

Two important aspects of recreational mathematics—the feel-good factor with positive emotions and the pedagogy—are interweaved to a significant extent. The boundary between recreational and serious mathematics is fuzzy. Due to pedagogical nature, recreational mathematics is as old as mathematics itself. In fact, "the history of mathematical recreations cannot be separated from the history of mathematics" (Chemla, 2014). In modern era, the first book, with recreational mathematics on its title, was published in 1624 (Heefer, 2004). Furthermore, games and mechanical puzzles are also related to recreational mathematics. It is interesting to note that various games of mathematical nature seem to be as ancient as our civilization.

These discussions broadly outline the conventional scope of recreational mathematics. However, due to variations in personal taste, the content may vary. As Trigg (1978) has pointed out, "Recreational tastes are highly individualized, so no classification of particular mathematical topics as recreational or not is likely to gain universal acceptance." I take this opportunity to select those subjects that seem recreational to me and include in this series.

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