EN 206: Power Electronics and Machines

Inverters

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Voltage Source Converters

- The fabricated AC voltage is not affected by load
- Applications:
  - AC Motor drives, Un interruptible power supply (UPS)
  - Induction heating, Power conversion from PV array and fuel cell
  - Static Var Compensator, Static Var Generator, Active harmonic fileter
- The power semi conductor devices are always forward biased due to dc supply voltage.
- GTO, BJT, IGCT, Power MOSFET, IGCT are suitable self controlled, forward or assymetric blocking devices
- Feed back diode is always connected across switch for free reverse flow of current.
VSC - General category

- **Pulse Width Modulated Inverters**
  - Input DC is essentially constant
  - Output voltage magnitude and frequency is controlled
  - Achieved using Pulse Width Modulation Technique

- **Square Wave Inverter**
  - Input DC is controlled to control output voltage magnitude
  - Inverter can control only frequency of output voltage
  - Output voltage waveform is similar to square wave.

- **Single phase inverter with voltage cancellation**
  - Input DC is essentially constant
  - Voltage cancellation technique is applicable for single phase inverters only.
Inductive load is connected between point 'a' and the centre point '0' of a split capacitor power supply.

Q1 and Q2 are closed alternately for $\pi$ angle to generate square wave output voltage.

$V_{ao}$ oscillates between $+0.5V_d$ and $-0.5V_d$. 
Single Phase inverter – Half bridge

- Snubber circuit is not shown for simplicity
- Short gap or lock out time $t_d$ is maintained to prevent any short circuit or shoot-through fault due to turn-off switching delay
- When supply voltage and current are of same polarity power is transferred from dc to ac or else power is fed back to source
- Average power flows from source to the load
Single Phase Full Bridge Inverter - H Bridge

- Split capacitor may not be required
- Q1Q2 and Q3Q4 are operated in pairs and switched alternately to generate square wave output voltage of amplitude $V_d$
- Feed back current flows through D1D2 and D3D4
- Both diodes are designed to withstand supply voltage $V_d$.
- H-Bridge inverters are used in four quadrant operation
Voltage Control using Phase Shift

\[ V_{ao} \]

\[ V_{bo} \]

\[ V_{ab} \]

\[ Q_1 \]

\[ Q_4 \]

\[ Q_3 \]

\[ Q_2 \]
Voltage Control – Phase Shift

- The output line voltage $V_{ab} = V_{a0} - V_{b0}$ is a quasi-square wave of pulse width “$\phi$”, which can control the fundamental component of output voltage.
- Assuming a typical lagging load current with perfect filtering:
  - Q1, Q2 conducting
    - Active mode with positive voltage and current
  - Q1, D3 conducting
    - Free wheeling mode with positive current
  - D3, D4 conducting
    - Feedback mode with positive current
  - Q3, Q4 conducting
    - Active mode with negative current and negative voltage
  - Q4, D2 conducting
    - Free wheeling with negative current
  - D1, D2 conducting
    - Feedback mode with negative current
Three Phase Inverter

Induction Motor Fed From AC Drive
Three Phase Square Wave Inverter - Waveform
Three Phase Inverter - Harmonic Spectrum

\[ \frac{\hat{V}_{LLh}}{V_d} \]

Harmonics of \( f_1 \)
Three Phase Inverter - Analysis

- \[ V_{LL,1(rms)} = \sqrt{\frac{3}{2}} \times \frac{4}{\pi} \times \frac{V_d}{2} = 0.78V_d \]
- \( V_{LL} \) does not depend on load condition and contains harmonics due to switching.
- \[ V_{LL,1(rms)} = 0.78V_d/h \] where, \( h = 6n \pm 1 \)
- It is not possible to control output voltage by using voltage cancellation technique in three phase inverter
- The period of conduction of each switch is determined by the power factor of the load
- Harmonic Spectrum
  - Even and Triplen harmonics are not present
  - PWM switching result in small ripple current
Pulse Width Modulation (PWM)

- Definition:??
- PWM inverters are becoming more popular for control of industrial drives due to advances in solid-state power devices and microprocessors.
- Frequency and magnitude of voltage and current of the motor can be controlled.
- Types: *Hysteresis PWM, Sine triangular PWM* and *space vector PWM*.
Sine Triangular PWM (SPWM)

(a)

(b)

(c)

\[
\frac{v_{A0}}{V_d/2}
\]

\[
m_a = 0.8, m_f = 15
\]
Pulse Width Modulation

Sine Triangular PWM (SPWM)

- Peak amplitude of the fundamental frequency component is $m_a$ times \( \frac{V_d}{2} \).
- The harmonics in the inverter output voltage waveform appear as side bands, centered around the switching frequency.
- The harmonics are given by $f_h = (jm_f \pm k)f_1$.
- For odd values of $j$, the harmonics exist only for even values of $k$.
- For even values of $j$, the harmonics exist only for odd values of $k$. 
Single Phase SPWM - Harmonic Analysis

Harmonics of $V_{Ao}$ for a large $m_f$. $V_{Ao,h}/\frac{V_d}{2} = V_{AN,h}/\frac{V_d}{2}$ is tabulated as a function of $m_a$.

<table>
<thead>
<tr>
<th>$m_a$</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.2</td>
<td>0.4</td>
<td>0.6</td>
<td>0.8</td>
<td>1.0</td>
</tr>
<tr>
<td>$m_f$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m_f \pm 2$</td>
<td>1.242</td>
<td>1.15</td>
<td>1.006</td>
<td>0.818</td>
<td>0.601</td>
</tr>
<tr>
<td></td>
<td>0.016</td>
<td>0.061</td>
<td>0.131</td>
<td>0.220</td>
<td>0.318</td>
</tr>
<tr>
<td>$m_f \pm 4$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$2m_f \pm 1$</td>
<td>0.19</td>
<td>0.326</td>
<td>0.37</td>
<td>0.314</td>
<td>0.181</td>
</tr>
<tr>
<td></td>
<td>0.016</td>
<td>0.024</td>
<td>0.071</td>
<td>0.139</td>
<td>0.212</td>
</tr>
<tr>
<td>$2m_f \pm 3$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.016</td>
<td>0.044</td>
<td>0.123</td>
<td>0.203</td>
<td>0.157</td>
</tr>
<tr>
<td>$3m_f \pm 2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$3m_f \pm 4$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$3m_f \pm 6$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$4m_f \pm 1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$4m_f \pm 3$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$4m_f \pm 5$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$4m_f \pm 7$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Sine Triangular PWM (SPWM)

- By choosing $m_f$ as odd integer results in odd symmetry as well as half wave symmetry with time origin.
- Only odd harmonics are present and the even harmonics disappear from the output waveform.
- Coefficients of the sine series in the fourier analysis are finite.
- Coefficients of the cosine series are zero.
- Switching losses are proportional to switching frequency. Higher switching frequency $\Rightarrow$ more losses.
- Small value of $m_f \leq 21$
  - Synchronous PWM
    - $m_f$ should be an integer otherwise, possibility of sub harmonics
    - $m_f$ should be an odd integer
- Harmonics due to over modulation ($ma > 2.5$)
SPWM - Over modulation

\[ \frac{(V_{Ao})_h}{(V_d/2)} \]

- \( m_a = 2.5 \)
- \( m_f = 15 \)

Harmonic \( h \)
Bi-Polar Voltage Switching

CIRCUIT DIAGRAM

- The diagonally opposite switches ($T_{A+}$ and $T_{B-}$) and ($T_{A-}$, $T_{B+}$) are switches as pairs.
- The output of inverter leg B is negative of the leg A output.
- When $T_{A+}$ is ON, $V_{A0} = +\frac{V_d}{2}$ and when $T_{B-}$ is ON $V_{B0} = -\frac{V_d}{2}$
The peak of the fundamental frequency component is \( V_{0a} = m_a V_d \).

The voltage switches between \( V_d \) and \( +V_d \) and hence called as bipolar voltage switching.
Bi-Polar Voltage Switching

- The lowest harmonics appear as side band of twice the switching frequency.
- Harmonic component of switching frequency disappear
Bi-Polar Voltage Switching

- The output current circulates in a loop through $T_{A+}$ and $D_{B+}$ or $D_{A+}$ and $T_{B+}$ depending on the direction of $i_0$.
- The output voltage changes between zero and $+V_d$ or zero and $V_d$ and hence names are unipolar voltage switching.
- The voltage jumps in output is limited to $V_d$ compared to $2V_d$ in the case of bipolar scheme.
Bi-Polar Voltage Switching

- The switches in the two legs of the full bridge are not switched simultaneously.
- Leg A and Leg B are controlled by comparing $V_{tri}$ with $V_{control}$ and $V_{control}$ respectively.
Uni-Polar Voltage Switching

Diagram showing the pulse width modulation process, with labeled components and waveforms for different voltage levels and time intervals.

- $V_{AN}$: Waveform for $V_{AN}$
- $V_{BN}$: Waveform for $V_{BN}$
- $V_{AN} - V_{BN}$: Difference waveform
- $T_B$: Time interval
- $T_A$: Time interval
- $v_{tri}$: Triangular waveform
- $v_{control}$: Control waveform
- $V_d$: Voltage level

Description:
- The diagram illustrates the switching process in a uni-polar voltage inverter, showing the relationship between control signals and the resulting output voltage waveforms.
- It highlights the switching instants and the corresponding voltage levels, emphasizing the pulse width modulation technique used in inverters.

Analysis:
- The diagram effectively visualizes how the control signals are used to modulate the output voltage, demonstrating the fundamental principles of pulse width modulation in inverters.
- Understanding these waveforms is crucial for designing and optimizing inverter systems, particularly in applications requiring precise control over voltage levels and switching times.
Uni-Polar Voltage Switching Pattern

- **Control logic - Switching Pattern**
  - $V_{\text{control}} > V_{\text{tri}}$; $T_{A+}$ on and $V_{AN} = V_d$
  - $V_{\text{control}} < V_{\text{tri}}$; $T_{A-}$ on and $V_{AN} = 0$
  - $-V_{\text{control}} > V_{\text{tri}}$; $T_{B+}$ on and $V_{BN} = V_d$
  - $-V_{\text{control}} < V_{\text{tri}}$; $T_{B-}$ on and $V_{BN} = 0$

- **Combination of switch on states and corresponding voltages**
  - $T_{A+}$ and $T_{B-}$ on, $V_{AN} = V_d$, $V_{BN} = 0$, $V_0 = V_d$
  - $T_{A-}$ and $T_{B+}$ on, $V_{AN} = 0$, $V_{BN} = V_d$, $V_0 = -V_d$
  - $T_{A+}$ and $T_{B+}$ on, $V_{AN} = V_d$, $V_{BN} = V_d$, $V_0 = 0$
  - $T_{A-}$ and $T_{B-}$ on, $V_{AN} = 0$, $V_{BN} = 0$, $V_0 = 0$

- When all the upper switches are on simultaneously, the output voltage is zero. The same is true for lower switches.
Three Phase SPWM - Switching Pattern
Three Phase SPWM - Harmonic Analysis

\[
\frac{(V_{LL})_h}{V_d}
\]

- \(m_a = 0.8, m_f = 15\)

Harmonics of \(f_1\):
- \(m_f\)
- \((m_f + 2)\)
- \((2m_f + 1)\)
- \(3m_f\)
- \((3m_f + 2)\)
Three Phase SPWM - Harmonic Analysis

Harmonics of $V_{LL}$ for a large and odd $m_f$ that is multiple of 3.

<table>
<thead>
<tr>
<th>$m_a$</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.122</td>
<td>0.245</td>
<td>0.367</td>
<td>0.49</td>
<td>0.612</td>
</tr>
<tr>
<td>$m_f \pm 2$</td>
<td>0.010</td>
<td>0.037</td>
<td>0.080</td>
<td>0.135</td>
<td>0.195</td>
</tr>
<tr>
<td>$m_f \pm 4$</td>
<td>0.116</td>
<td>0.2</td>
<td>0.227</td>
<td>0.192</td>
<td>0.111</td>
</tr>
<tr>
<td>$2m_f \pm 1$</td>
<td>0.027</td>
<td>0.085</td>
<td>0.124</td>
<td>0.108</td>
<td>0.038</td>
</tr>
<tr>
<td>$2m_f \pm 5$</td>
<td>0.027</td>
<td>0.007</td>
<td>0.029</td>
<td>0.064</td>
<td>0.096</td>
</tr>
<tr>
<td>$3m_f \pm 2$</td>
<td>0.1</td>
<td>0.096</td>
<td>0.005</td>
<td>0.064</td>
<td>0.042</td>
</tr>
<tr>
<td>$3m_f \pm 4$</td>
<td>0.027</td>
<td>0.096</td>
<td>0.021</td>
<td>0.051</td>
<td>0.073</td>
</tr>
<tr>
<td>$4m_f \pm 1$</td>
<td>0.021</td>
<td>0.01</td>
<td>0.01</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>$4m_f \pm 5$</td>
<td>0.027</td>
<td>0.007</td>
<td>0.029</td>
<td>0.064</td>
<td>0.096</td>
</tr>
<tr>
<td>$4m_f \pm 7$</td>
<td>0.1</td>
<td>0.096</td>
<td>0.005</td>
<td>0.064</td>
<td>0.042</td>
</tr>
</tbody>
</table>

Note: $V_{LL,h}/V_d$ are tabulated as a function of $m_a$ where $V_{LL,h}$ are the rms values of the harmonic voltages.
- Triangular wave form is compared to three sinusoidal waveforms $120^\circ$ apart to generate pulses for the bridge circuit.
- DC voltage present in the phase voltage gets canceled out in the line voltages.
- In the case of three phase inverter, only line voltages are of importance.
- The phase difference between $m_f^{th}$ harmonic is zero between two phases and hence cancel out in line voltage, $m_f$ is odd integer and multiple of 3.
- Dominant harmonics present in single phase inverter are eliminated from the line-line voltage of a three phase inverter.
- The peak value of fundamental of one of the leg of inverter is $\hat{V}_{AN,1} = m_a \frac{V_d}{2}$ and the line-line voltage is given by $V_{LL,1(rms)} = \sqrt{\frac{3}{2}}(\hat{V}_{AN,1}) = 0.612m_aV_d$.
Comparison - PWM techniques

\[ \frac{V_{LL1}}{V_d} = \frac{\sqrt{6}}{\pi} = 0.78 \]

\[ \frac{\sqrt{3}}{2\sqrt{2}} = 0.612 \]

For \( m_f = 15 \):

- Linear
- Overmodulation
- Square-wave
Space Vector PWM

\[
\begin{bmatrix}
V_{ab} \\
V_{bc} \\
V_{ca} \\
V_a \\
V_b \\
V_c
\end{bmatrix} = \begin{bmatrix}
1 & -1 & 0 \\
0 & 1 & -1 \\
-1 & 0 & 1 \\
2 & -1 & -1 \\
-1 & 2 & -1 \\
-1 & -1 & 2
\end{bmatrix} \begin{bmatrix}
a \\
b \\
c \\
a \\
b \\
c
\end{bmatrix}
\]

When the upper switch is ON the corresponding lower switch is OFF.
The state of the switch is sufficient to evaluate the output voltage.
There are eight possible combinations for on/off state of the upper switches.
## SVPWM - On/Off state and Corresponding Output

<table>
<thead>
<tr>
<th>Switch State</th>
<th>Phase Voltage</th>
<th>Line Voltage</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 0 0</td>
<td>0 0 0</td>
<td>0 0 0</td>
</tr>
<tr>
<td>1 0 0</td>
<td>2/3 -1/3 -1/3</td>
<td>1 0 -1</td>
</tr>
<tr>
<td>1 1 0</td>
<td>1/3 1/3 -2/3</td>
<td>0 1 -1</td>
</tr>
<tr>
<td>0 1 0</td>
<td>-1/3 2/3 -1/3</td>
<td>-1 1 0</td>
</tr>
<tr>
<td>0 1 1</td>
<td>-2/3 1/3 1/3</td>
<td>-1 0 1</td>
</tr>
<tr>
<td>0 0 1</td>
<td>-1/3 -1/3 2/3</td>
<td>0 -1 1</td>
</tr>
<tr>
<td>1 0 1</td>
<td>1/3 -2/3 1/3</td>
<td>1 -1 0</td>
</tr>
<tr>
<td>1 1 1</td>
<td>0 0 0</td>
<td>0 0 0</td>
</tr>
</tbody>
</table>
Sample calculations for Hexagon

\[
\begin{bmatrix}
V_a \\
V_b \\
V_c
\end{bmatrix} = \frac{1}{3} V_{dc} \begin{bmatrix}
2 & -1 & -1 \\
-1 & 2 & -1 \\
-1 & -1 & 2
\end{bmatrix}
\begin{bmatrix}
a \\
b \\
c
\end{bmatrix}
\]

\[
T_{abc-dq} = \sqrt{\frac{2}{3}} \begin{bmatrix}
1 & -1/2 & -1/2 \\
0 & \sqrt{3}/2 & -\sqrt{3}/2
\end{bmatrix}
\begin{bmatrix}
V_a \\
V_b \\
V_c
\end{bmatrix}
\]

\[
\begin{bmatrix}
d \\
q
\end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix}
1 & -1/2 & -1/2 \\
0 & \sqrt{3}/2 & -\sqrt{3}/2
\end{bmatrix} \frac{1}{3} V_{dc}
\begin{bmatrix}
2 & -1 & -1 \\
-1 & 2 & -1 \\
-1 & -1 & 2
\end{bmatrix}
\begin{bmatrix}
a \\
b \\
c
\end{bmatrix}
\]

For vector \( U_{100} \): Substituting \( a=1, b=0, c=0 \) gives,

\[
\begin{bmatrix}
d \\
q
\end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix}
1 & -1/2 & -1/2 \\
0 & \sqrt{3}/2 & -\sqrt{3}/2
\end{bmatrix} \frac{1}{3} V_{dc}
\begin{bmatrix}
2 & -1 & -1 \\
-1 & 2 & -1 \\
-1 & -1 & 2
\end{bmatrix}
\begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix}
\]

\[
\begin{bmatrix}
d \\
q
\end{bmatrix} = \begin{bmatrix}
\sqrt{(2/3)} \\
0
\end{bmatrix}
\]
SVPWM - Table for Space Vector

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>$V_a$</th>
<th>$V_b$</th>
<th>$V_c$</th>
<th>$V_{ab}$</th>
<th>$V_{bc}$</th>
<th>$V_{ca}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0000</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$U_{100}$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>2/3</td>
<td>-1/3</td>
<td>-1/3</td>
<td>1</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>$U_{110}$</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1/3</td>
<td>1/3</td>
<td>-2/3</td>
<td>0</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>$U_{010}$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>-1/3</td>
<td>2/3</td>
<td>-1/3</td>
<td>-1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$U_{011}$</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>-2/3</td>
<td>1/3</td>
<td>1/3</td>
<td>-1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$U_{001}$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>-1/3</td>
<td>-1/3</td>
<td>2/3</td>
<td>0</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>$U_{101}$</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1/3</td>
<td>-2/3</td>
<td>1/3</td>
<td>1</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>$U_{111}$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The generated or reference voltage shall lie in the hexagon formed by the above vectors.
There are total of 6 sectors in which the reference voltage $U_{out}$ shall belong to.

If the reference output voltage magnitude and angle is given, then $|U_{out}|$ and $\alpha$ can be computed. Where $\alpha$ is angle between $U_{out}$ and $U_x$.

From $|U_{out}|$ and $\alpha$, the sector of reference voltage $U_{out}$ can be easily computed.

Time period for which the vectors shall operate is given by:

$$T_1 + T_2 + T_0 = T_{pwm}$$
$$T_1 = \sqrt{(2)} T_{pwm} |U_{out}| \cos(\alpha + 30^0)$$
$$T_2 = \sqrt{(2)} T_{pwm} |U_{out}| \sin(\alpha)$$
$$T_{pwm} U_{out} = T_1 U_x + T_2 U_{(x\pm60)} + T_0 (0000 \ or \ 0111)$$
SVPWM - Switching Direction
The maximum value of $U_{out}$ is the shortest radius of the envelope.

The maximum rms value of line-line voltage is $\frac{V_d}{\sqrt{2}}$ and the maximum rms value of phase voltage is $\frac{V_d}{\sqrt{6}}$ which is $\frac{2}{\sqrt{3}}$ times higher than that of sine triangular PWM technique.

If the motor is rated for $V_{rms}$ (three phase L-L) then the dc bus requires shall be $V_d = \sqrt{2} \times V_{rms}$

$U_x$ can be basic closest space vector on either side of $U_{out}$. $U_{x+60}$ (or $U_{x-60}$) is basic space vector on the opposite side.
SVPWM - Switching

- $T_1$ represents component on $U_x$ and $T_2$ represent component on the other vector. Each PWM channel switches twice per every PWM period except when the duty cycle is 0% or 100%.
- There is a fixed switching order among the three PWM channels for each sector.
- Every PWM period starts and ends with $O_{000}$; The amount of $O_{000}$ inserted is the same as that of $O_{111}$ in each PWM period.
- The above is applicable for symmetric PWM.
Pulse Width Modulation

Three Phase SPWM - Switching Pattern

Prof. Doolla (DESE)
Three Phase SPWM - Inverter Output