THERMOECONOMIC OPTIMIZATION OF COMBINED-CYCLE POWER PLANTS

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ABSTRACT

Introduction of irreversibility in design and operation of any power plant is essential due to constraints in finite resources (such as finite time of operation, finite size of the plant, finite capital investment, finite power production, etc.). The thermoeconomic optimization of a combined-cycle power plant, comprising an arbitrary number of internally irreversible Carnot-like heat engines, are studied in this paper considering the finite resource constraints. The efficiency of the multistage endoreversible combined-cycle power plant corresponding to maximum power production or minimum operating cost are observed to be identical to those of a single endoreversible heat engine under same operating conditions. Increase in number of stages reduces the power production but increases the total annualized cost of the plant. The inventory control of heat exchanger surface area or their thermal conductance and the direction for heat transfer augmentation to get maximum benefits are also discussed in this work. The flexibility in selecting different working fluids at different operating pressures are identified for optimal design and operation of the combined-cycle power plant.

key words: thermoeconomics / combined-cycle / irreversible Carnot engine / design flexibility / heat exchanger inventory.

INTRODUCTION

In classical thermodynamics, Carnot heat engines operate reversibly with an efficiency given by, $\eta_C = 1 - T_{min}/T_{max}$ and produces no power as it requires infinite operation time or alternately involves infinite costs to provide infinite heat exchanger surface area. The optimal performance of an internally reversible or endoreversible Carnot-like heat engine, operating in finite time with finite size of the plant, corresponds to its operating efficiency $\eta_{CA} = 1 - \sqrt{T_{min}/T_{max}}$, and delivers maximum power (Curzon and Ahlborn, 1975). The efficiency of cost-optimally operated Carnot-like endoreversible heat engine is given by (Bandyopadhyay and Bera, 1996)

$$\eta_{opt} = 1 - \sqrt{\frac{T_{min}}{T_{max}}\left(\frac{g_3 - g_1}{g_2 + g_3}\right)} \qquad (1)$$

where the g's are the different cost coefficients.

The efficiency and the work output of a multistage reversible power plant are identical to those of a single reversible heat engine. Such reversible combined-cycle power plants produce no power. Consequently, studies on the thermodynamic performances of 2-stage endoreversible combined-cycle power plants have been reported (Rubin and Anderson, 1982; Wu *et al.*, 1992; Bejan, 1995; Özkaynak, 1995; Chen and Wu, 1996). Rubin and Andersen (1982) and Bejan (1995) noted that the efficiency of the optimal endoreversible combined-cycle power plant is same as a single endoreversible heat engine delivering maximum power when operating between the same temperature limits, although the power output of the 2-stage combined-cycle is considerably less than that of a single heat engine.

In this paper, thermoeconomic performance of an *n*-stage internally irreversible combined-cycle power plant is presented. The effects of distribution of heat exchanger surface area, their thermal conductance and heat transfer augmentation on thermoeconomic aspects of the power plant are also addressed. The design and operation philosophy are extended to include cogeneration systems.

COMBINED-CYCLE POWER PLANT

The schematic diagram of a multistage combined-cycle power plant is shown in Figure 1. Each heat engine in the system is combined through the heat exchangers. The immediate bottoming engine receives heat rejected from the topping engine without any loss. Thus, *i*-th heat engine receives Q_i amount of heat from the (i–1)-th topping heat engine and rejects

Q_{i+1} amount of heat to the (i+1)-th bottoming heat engine. The operating temperatures of the i-th engine are $T_{i,1}$ and $T_{i,2}$, respectively and its power output is W_i with efficiency η_i. Only the first and the last heat engines are exposed to the hot ($T_{\max}$) and cold ($T_{\min}$) reservoirs, respectively. The heat engines are assumed to operate continuously under steady-state condition. Assuming linear (i.e., Newtonian) heat transfer, heat transferred through the i-th heat exchanger may be written in terms of its thermal conductance, K_i.

$$Q_i = K_i\left(T_{i-1,2} - T_{i,1}\right) \tag{2}$$

Thermal conductance is the product of exchanger surface area (A_i) with overall heat transfer coefficient (U_i) of the exchanger (i.e., $K_i = U_i A_i$). All the engines are assumed to be internally irreversible. Therefore Clausius inequality for steady-state cyclic operation may be written as,

$$\frac{Q_i}{T_{i,1}} - \frac{Q_{i+1}}{T_{i,2}} \le 0$$

$$\Rightarrow \frac{Q_i}{T_{i,1}} - \phi_i \frac{Q_{i+1}}{T_{i,2}} = 0 \tag{3}$$

Equation (3) defines the irreversibility factor ($\phi_i \le 1$) that measures the thermodynamic irreversibilities such as friction, pressure drops, internal heat transfer over finite temperature difference, etc. For endoreversible or internally reversible heat engine, ϕ_i is unity. Employing these irreversibility criteria, Q_{i+1} may be expressed in terms of heat input to the power plant as $Q_{i+1} = Q_1 \prod_{j=1}^{i}\left(\tau_j/\phi_i\right)$, where τ_i is the ratio of the cold to hot operating temperature of i-th heat engine ($\tau_i = T_{i,2}/T_{i,1}$). Energy balance on each heat engine yields the power produced by the engine.

$$W_i = Q_i - Q_{i+1} \tag{4}$$

Efficiency of an individual engine may be expressed as $\eta_i = W_i/Q_i = 1 - \tau_i/\phi_i$.

The overall efficiency of the combined-cycle power plant is given by

$$\eta = \frac{W(n)}{Q_1} = \frac{\sum_{i=1}^{n} W_i}{Q_1}$$

$$= \frac{\sum_{i=1}^{n}\left(Q_i - Q_{i+1}\right)}{Q_1}$$

$$= \frac{Q_1 - Q_{n+1}}{Q_1} = 1 - \prod_{i=1}^{n}\left(\frac{\tau_i}{\phi_i}\right) \tag{5}$$

Therefore, the overall efficiency of the power plant is independent of the heat exchanger sizes, but depends on the operating temperature ratio and the irreversibility factor of the individual heat engine. Combining the above equations the heat absorbed, the heat rejected, and the power output of the overall combined power plant may be obtained, respectively, as

$$Q_{in} = Q_1 = \frac{1}{\sum_{i=1}^{n+1} \frac{\phi_i}{K_i}} \left[\frac{T_{\max} \prod_{i=1}^{n} \tau_i - T_{\min}}{\prod_{i=1}^{n} \left(\frac{\tau_i}{\phi_i} \right)} \right] \quad (6)$$

$$Q_{out} = Q_{n+1} = \frac{T_{\max} \prod_{i=1}^{n} \tau_i - T_{\min}}{\sum_{i=1}^{n+1} \frac{\phi_i}{K_i}} \quad (7)$$

$$W(n) = \sum_{i=1}^{n} W_i = \frac{\left[\prod_{i=1}^{n} \left(\frac{\phi_i}{\tau_i} \right) - 1 \right] \left(T_{\max} \prod_{i=1}^{n} \tau_i - T_{\min} \right) 1}{\sum_{i=1}^{n+1} \frac{\phi_i}{K_i}} \quad (8)$$

To make these expressions elegant, ϕ_{n+1} is assumed to be unity without attaching any physical significance to it. Irreversibility factor of the *i*-th engine is associated with the thermal conductance of its hot side heat exchanger (i.e., the *i*-th heat exchanger). Therefore, the irreversibility factor may be called the associated irreversibility factor of the *i*-th heat exchanger. The associated irreversibility factor for the last exchanger is unity. These results may be derived by induction with simple but tedious algebraic manipulations.

Recognizing that the efficiency and the power output of the overall plant must be positive, the product of the operating temperature ratios of all heat engines, should lie between the product of the irreversibility factors and cold to hot reservoir temperature ratio (i.e., $1 \geq \prod_{i=1}^{n} \phi_i \geq \prod_{i=1}^{n} \tau_i \geq T_{\min}/T_{\max}$). From equation (8), it may be concluded that no power is produced by the combined-cycle power plant when $\prod_{i=1}^{n} \tau_i$ is either $\prod_{i=1}^{n} \phi_i$ or $T_{\min}/T_{\max}$. Note that, $\prod_{i=1}^{n} \tau_i$ equals $\prod_{i=1}^{n} \phi_i$ corresponding to the case of thermal short-circuit or total thermal leakage when all heat is leaked past the plant and on the other hand, the other limit corresponds to the thermal open-circuit limit. The optimal is located somewhere within these two limits. The condition for maximum power ($\partial W(n)/\partial \tau_i = 0$) corresponds to

$$\prod_{i=1}^{n} \tau_i = \sqrt{\frac{T_{\min}}{T_{\max}} \prod_{i=1}^{n} \phi_i} \quad (9)$$

and the corresponding efficiency of the overall power plant is

$$\eta_{MP} = 1 - \sqrt{\frac{T_{\min}}{T_{\max}} \prod_{i=1}^{n} \frac{1}{\phi_i}} \quad (10)$$

The efficiency of the power plant increases with increasing maximum temperature limit of the plant and with decreasing internal irreversibility of the individual heat engines. Thus, for multistage endoreversible combined-cycle power plant ($\prod_{i=1}^{n} \phi_i = 1$), efficiency at maximum power is the same as Curzon-Ahlborn (1975) efficiency of a single heat engine delivering maximum power. Equivalent result for 2-stage endoreversible combined-cycle has been reported by Rubin and Andersen (1982), Bejan (1995) as well as Chen and Wu (1996).

As $\tau_i \geq \prod_{i=1}^{n} \tau_i$, the overall efficiency of the power plant is more than the efficiency of the individual heat engine it consists off. If the working temperature ratios of some heat engines are restricted due to material and working fluid constraints, combined-cycle power plant will perform better if the operating temperature range is increased. As for example, the operating temperature of a steam-Rankine cycle is generally restricted to 550°C. The power output and the operating efficiency may greatly be improved by combining with a Joule-Brayton topping cycle (1100° - 600° C). The individual operating temperature ranges (1100° - 600° C and 550° - 40° C) are smaller than the whole power plant (1100° - 40° C) and hence benefits like higher efficiency, higher power output, reduced emission levels are achieved.

Interestingly, the criterion for maximum power production of the multistage combined-cycle power plant is collective in nature. The ratio of the operating temperature of each heat engine may be chosen arbitrarily satisfying equation (9). This allows the designer to select different working fluids at different operating pressures and thus, provides enough flexibility in design. Throughout this study this fact prevails and instead of indicating product of working temperature ratio of different engines, it will be denoted as a single term $\tau = \prod_{i=1}^{n} \tau_i$, where τ is the effective temperature ratio of the multistage combined-cycle power plant.

The maximum power output of the plant is expressed as:

$$W_{\max}(n) = \frac{\left(\sqrt{T_{\max} \prod_{i=1}^{n} \phi_i} - \sqrt{T_{\min}}\right)^2}{\sum_{i=1}^{n+1} \frac{\phi_i}{K_i}} \tag{11}$$

The maximum power is inversely proportional to the total thermal resistance offered by the heat exchangers and may be increased by investing on more thermal conductance and by decreasing internal irreversibility of the individual heat engine $(\phi_i \rightarrow 1)$.

Maximization of power for given total conductance

The power produced by the multistage combined-cycle power plant can further be optimized for given total thermal conductance of the power plant ($K = \sum_{i=1}^{n+1} K_i$, constant). Denoting β_i $(= K_i / K)$ as the fractional thermal conductance of *i*-th exchanger (that is, hot side exchanger of the heat engine *i*) equation (11) may be expressed as

$$\frac{W_{\max}(n)}{K} = \frac{\left(\sqrt{T_{\max}\prod_{i=1}^{n}\phi_i} - \sqrt{T_{\min}}\right)^2}{\sum_{i=1}^{n+1}\frac{\phi_i}{\beta_i}} = \frac{\left(\sqrt{T_{\max}\prod_{i=1}^{n}\phi_i} - \sqrt{T_{\min}}\right)^2}{\sum_{i=1}^{n}\frac{\phi_i}{\beta_i} + \left(1-\sum_{i=1}^{n}\beta_i\right)^{-1}}$$

2 (12)

Thus, for the power plant having size constraint, the power produced per unit of total thermal conductance is given by equation (12). Maximization $[\partial W_{\max}(n)/\partial\beta_i = 0]$ of output power for given total thermal conductance, leads to $\beta_i = \beta_{n+1}\sqrt{\phi_i}$. Since $\sum_{i=1}^{n+1}\beta_i = 1$, therefore

$$\beta_i = \frac{\sqrt{\phi_i}}{\sum_{j=1}^{n+1}\sqrt{\phi_j}} \tag{13}$$

This result signifies that the thermal conductance should be distributed proportional to the square root of the associated irreversibility factors. For endoreversible power plant the thermal conductance should be distributed equally among all the exchangers. Corresponding maximum power output is given by

$$W_{\max,\max(K)}(n) = K\left[\left(\sqrt{T_{\max}\prod_{i=1}^{n}\phi_i} - \sqrt{T_{\min}}\right) / \left(\sum_{i=1}^{n+1}\sqrt{\phi_i}\right)\right]^2$$

3(14)

Maximum power output of a multistage combined-cycle power plant and single stage heat engine (with $n = 1$), working between same maximum and minimum temperature, may be compared to obtain

$$\frac{W_{\max,\max(K)}(n)}{W_{\max,\max(K)}(1)} = \left(\frac{1+\sqrt{\phi_1}}{\sum_{i=1}^{n+1}\sqrt{\phi_i}}\right)^2 \le 1 \tag{15}$$

Therefore, the power production reduces as the number of stages increases. This is because of decrease in effective heat exchanger size (as the total thermal conductance is distributed among $n+1$ exchangers), the introduction of irreversibility by many heat exchangers and heat engines, instead of just one of them.

Maximization of power for given total exchanger area

Instead of given total thermal conductance, total exchanger area of the power plant may also be given to be constant ($A = \sum_{i=1}^{n+1} A_i$, constant). Equation (11) may be modified to

$$\frac{W_{\max}(n)}{A} = \frac{\left(\sqrt{T_{\max}\prod_{i=1}^{n}\phi_i} - \sqrt{T_{\min}}\right)^2}{A\sum_{i=1}^{n+1}\frac{\phi_i}{U_i A_i}}$$

$$= \frac{\left(\sqrt{T_{\max}\prod_{i=1}^{n}\phi_i} - \sqrt{T_{\min}}\right)^2}{\sum_{i=1}^{n}\frac{\phi_i}{U_i\alpha_i} + U_{n+1}^{-1}\left(1-\sum_{i=1}^{n}\alpha_i\right)^{-1}}$$

4 (16)

where α_i is the ratio of i-th exchanger surface area to the total exchanger area. Maximum power [$\partial W_{\max}(n)/\partial\alpha_i = 0$] for given total exchanger surface area, corresponds to

$$\alpha_i = \alpha_{n+1}\sqrt{\frac{\phi_i U_{n+1}}{U_i}}$$

$$= \left(\sum_{j=1}^{n+1}\sqrt{\frac{\phi_j}{U_j}}\right)^{-1}\sqrt{\frac{\phi_i}{U_i}} \quad 5 \qquad (17)$$

Therefore, the allocation of exchanger surface area is inversely proportional to the square root of the overall heat transfer coefficient of the exchanger and directly proportional to the square root of the associated irreversibility factor. The corresponding maximum power output is given by

$$W_{\max,\max(A)}(n) = A\left[\left(\sqrt{T_{\max}\prod_{i=1}^{n}\phi_i} - \sqrt{T_{\min}}\right)/\left(\sum_{i=1}^{n+1}\sqrt{\phi_i/U_i}\right)\right]^2$$

(18)

Comparing the maximum power output of multistage combined-cycle power plant and single stage heat engine, working between same maximum and minimum temperatures yields

$$\frac{W_{\max,\max(A)}(n)}{W_{\max,\max(A)}(1)} = \left(\frac{\sqrt{\frac{\phi_1}{U_1}} + \sqrt{\frac{1}{U_{n+1}}}}{\sum_{i=1}^{n+1}\sqrt{\frac{\phi_i}{U_i}}}\right)^2 \leq 1$$

(19)

Equation (19) shows a reduction in power for combined-cycle plants compared to the single stage heat engines. If all U_i's are equal, equations (18) and (19) reduce to equations (14) and (15), respectively.

Distribution of area among heat exchangers

The size constraint of the power plant may be put in two ways, constant total thermal conductance and constant total exchanger area (Bejan, 1995). The distribution of the exchanger area may be determined for these two constraints. When the total thermal conductance is constant, equation (13) leads to

$$\frac{A_i}{A_j} = \frac{U_j}{U_i}\sqrt{\frac{\phi_i}{\phi_j}} \qquad (20)$$

On the other hand, if the total exchanger area is constant, equation (17) suggests that

$$\frac{A_i}{A_j} = \sqrt{\frac{U_j\phi_i}{U_i\phi_j}} \quad 6 \qquad (21)$$

Thus, we conclude from equations (20) and (21) that larger area is to be distributed to that exchanger whose overall heat transfer coefficient to associated irreversibility factor is smaller. The ratio of overall heat transfer coefficient to associated irreversibility factor plays an important role in operating and designing the power plant optimally. As both the constraints suggest qualitatively the same thing, power plants with given constant exchanger area will be considered for the rest of this study.

Augmentation of heat exchangers

To enhance the power generated by the multistage power plant, heat exchangers are usually augmented to increase the overall heat transfer coefficient. Now the question is where should the augmentation effort and investment be directed to get the maximum benefit. To get an answer to this, we differentiate equation (18) with respect to U_j to obtain:

$$\frac{\partial\left[W_{\max,\max(A)}(n)\right]}{\partial U_j} \propto \sqrt{\frac{\phi_j}{U_j^3}} \qquad (22)$$

This suggests that the increase in power generation due to augmentation in the *i*-th exchanger is inversely proportional to 3/2 power of its overall heat transfer coefficient. Comparing this with performance of the *j*-th exchanger we get,

$$\frac{\partial\left[W_{\max,\max(A)}(n)\right]}{\partial U_i} \bigg/ \frac{\partial\left[W_{\max,\max(A)}(n)\right]}{\partial U_j}$$

$$= \sqrt{U_j^3\phi_i / U_i^3\phi_j} \qquad (23)$$

Now, $\partial\left[W_{\max,\max(A)}(n)\right] / \partial U_i \leq \partial\left[W_{\max,\max(A)}(n)\right] / \partial U_j$, whenever $\left(U_i^3 / \phi_i\right) \geq \left(U_j^3 / \phi_j\right)$. Therefore, augmentation of heat transfer coefficient should be directed towards that exchanger which has the lowest value of $\left(U_i^3 / \phi_i\right)$. Hence, for endoreversible combined-cycle power plants, augmentation efforts should be directed towards the exchanger with the lowest overall heat transfer coefficient.

THERMOECONOMIC DESIGN OF COMBINED-CYCLE

The efficiency of heat engines operating at minimum cost is typically higher than the efficiency corresponding to the maximum power condition (Bejan, 1993; De Vos, 1995; Bandyopadhyay and Bera, 1996). As the efficiency of the engine decreases from reversible Carnot limit, both power production and fuel consumption increase. This brings the engine to operate somewhere between the maximum power point and the reversible operating point with maximum efficiency. Thermoeconomic study for a single endoreversible heat engine have been presented by Bandyopadhyay and Bera (1996). Bera and Bandyopadhyay (1998) have studied the effect of combustion on the operating cost of endoreversible Otto and Joule-Brayton engine.

Let, g_1 be the unit cost of input energy (proportional to fuel cost), g_2 be the unit cost of heat rejection (depends cooling utility cost), and g_3 be the unit selling price of power produced. The operating cost of the plant is

$$\sigma_{op}(n) = g_1 Q_{in} + g_2 Q_{out} - g_3 W(n)$$
$$= \left(T_{\max}\tau - T_{\min}\right)\left[\left(g_2 + g_3\right)\tau - \left(g_3 - g_1\right)\prod_{i=1}^{n}\phi_i\right]\left[\tau\sum_{i=1}^{n+1}\left(\frac{\phi_i}{K_i}\right)\right]^{-1} \quad (24)$$

The economic design should be such that

$$\frac{T_{\min}}{T_{\max}} \le \tau \le \left(\frac{g_3 - g_1}{g_3 + g_2}\right)\prod_{i=1}^{n}\phi_i \quad (25)$$

Within this range the operating cost will be negative, that is, the operation of the power plant will be profitable. This criterion of feasible economic operation may be rapidly translated to calculate the minimum selling price of power.

Power production and cost effective operation are not the only criteria for designing a power plant. The life of the whole plant as well as the total investment must also be taken into account. If the annualized capital cost per unit of exchanger area is '*a*' and other annualized fixed cost is '*b*' then annualized capital cost may be expressed as

$$\sigma_{cap}(n) = b + \sum_{i=1}^{n+1} a\, A_i \quad (26)$$

Therefore, the *total annualized cost* (TAC) of the combined-cycle power plant is given by

$$\sigma_{tac}(n) = \sigma_{op}(n) + \sigma_{cap}(n)$$
$$= b + a\sum_{i=1}^{n+1} A_i + \left(T_{\max}\tau - T_{\min}\right)\left[\left(g_2 + g_3\right)\tau - \left(g_3 - g_1\right)\prod_{i=1}^{n}\phi_i\right]\left[\tau\sum_{i=1}^{n+1}\left(\frac{\phi_i}{U_i A_i}\right)\right]^{-1} \quad (27)$$

Minimizing TAC (corresponds to most profitable design) with respective to effective temperature ratio of the power plant and distribution of the heat exchanger area, the following is obtained :

$$\sigma_{tac,\min}(n) = b + aA - A\left[\left(\sqrt{\left(g_3 - g_1\right)T_{\max}\prod_{i=1}^{n}\phi_i} - \sqrt{\left(g_2 + g_3\right)T_{\min}}\right)\left(\sum_{i=1}^{n+1}\sqrt{\frac{\phi_i}{U_i}}\right)^{-1}\right]^2$$

7 (28)

The optimum effective temperature ratio is expressed as:

$$\tau_{opt} = \sqrt{\frac{T_{\min}}{T_{\max}}\left(\frac{g_3 - g_1}{g_3 + g_2}\right)\prod_{i=1}^{n}\phi_i} \quad (29)$$

The corresponding efficiency of an endoreversible combined-cycle power plant is given in equation (1) and the minimum operating cost is given by

$$\sigma_{op,\min}(n) = -A\left[\left(\sqrt{(g_3 - g_1)T_{\max}\prod_{i=1}^{n}\phi_i} - \sqrt{(g_2 + g_3)T_{\min}}\right)\left(\sum_{i=1}^{n+1}\sqrt{\frac{\phi_i}{U_i}}\right)^{-1}\right]^2 \quad (30)$$

This reduces to the expression for an endoreversible heat engine presented by Bandyopadhyay and Bera (1996). The cost optimal performance of multistage endoreversible combined power plant is no better than the cost optimal performance of single endoreversible Carnot or Otto or Joule-Brayton heat engine (Bera and Bandyopadhyay, 1998). It is noteworthy that the operating cost and hence the TAC increases with increasing number of stages. Whenever energy is free (that is, $g_1 = g_2 = 0$), the cost optimal designing and operating criterion translates to the condition of maximum power.

Design for Given Input Energy

For combined heat and power generation, inlet or outlet thermal energy or both may be governed by the process side demand. The First and the Second law analysis of a process, separate it into two non-interactive disjoint sections, viz., high temperature heat sink and low temperature heat source. The temperature at which this separation occurs is known as pinch point. The appropriate placement of the heat engine depends on the thermal and as well the power demand of the process (Bandyopadhyay, 1995). For 100% marginal efficiency (when each extra unit of thermal interaction translates to equal amount of power production) the heat engine should be placed entirely above or entirely below the process pinch. Note that the 100% marginal efficiency does not mean that the efficiency of the heat engine is unity. Whenever the heat engine is placed above the process pinch, high temperature heat sink of the process dictates the amount of heat rejection from the heat engine. On the other hand, when the engine is placed below the process pinch, the heat received by the engine is determined by the low temperature heat source of the process. However, the placement of heat engine entirely below the process pinch is most beneficiary (Bandyopadhyay, 1995). In case of given energy input, total exchanger area can be calculated from equation (6) and then combining with equation (27), we get

$$\sigma_{tac,Qin}(n) = b + aQ_{in}\prod_{i=1}^{n}(\tau_i/\phi_i)\left[\sum_{i=1}^{n+1}\left(\frac{\phi_i}{U_i\alpha_i}\right)\Big/\left(T_{\max}\tau - T_{\min}\right)\right] + Q_{in}\left[(g_2 + g_3)\prod_{i=1}^{n}(\tau_i/\phi_i) - (g_3 - g_1)\right] \quad (31)$$

Optimum TAC corresponds to the distribution of exchanger surface area as given in equation (17) and the effective temperature ratio as given by

$$\tau_{opt,Qin} = \frac{T_{\min}}{T_{\max}}\left[1+\sum_{i=1}^{n+1}\sqrt{\frac{a\phi_i}{T_{\min}U_i\left(g_3+g_2\right)}}\right] \quad (32)$$

Total exchanger area and the input energy Q_{in} are related as

$$A(n) = \frac{Q_{in}}{T_{\max}}\left[\left(\sum_{i=1}^{n+1}\sqrt{\frac{\phi_i}{U_i}}\right)^2 + \sum_{i=1}^{n+1}\sqrt{\frac{T_{\min}\left(g_3+g_2\right)\phi_i}{aU_i}}\right] \quad (33)$$

Ratio of exchanger surface area for a multistage combined-cycle power plant to that of single heat engine, working between same limits of temperature with same overall heat transfer coefficient, is observed to be more than unity whereas the efficiency ratio of the multistage combined-cycle to that of a single engine is observed to be less than unity [since, $A(n)/A(1) \geq 1$ but $\eta(n)/\eta(1) \leq 1$]. Though the invested exchange area increases with increase in number of stages, efficiency of the power plant and hence the power output decreases.

Similar results can easily be obtained for the case when outlet energy is given or the engine is placed above the process pinch.

CONCLUSIONS

Efficiency and power output of a multistage combined-cycle reversible power plant are same as those of single reversible heat engine. The efficiency of an endoreversible heat engine or multistage combined-cycle endoreversible power plant depends on their operating characteristics. The efficiency of a multistage combined-cycle endoreversible plant is same as the Curzon-Ahlborn efficiency [equation (10)] when producing maximum power or is equal to the efficiency given by equation (1) when operated with minimum operating cost. The power generation of the plant decreases with increasing number of stages and the other way round for TAC of the plant. The thermoeconomic performances of a combined-cycle may be greatly improved by increasing the operating temperature range of the plant (for example, potassium/steam cycle; gas turbine/steam turbine cycle; MHD generator/steam cycle; thermoelectric generator/steam cycle; etc.).

The distribution of the thermal conductance or the exchanger surface area plays an important role in management of heat exchanger inventory. Both these criteria qualitatively suggest that the maximum area should be attached to the exchanger whose overall heat transfer coefficient is the least. In fact, the exchanger with the least overall heat transfer coefficient should be augmented first.

Most interestingly, the underlying flexibility of selection of working fluids and their operating pressure is identified in this paper. Cost optimal design of multistage combined-cycle power plant is also discussed in this paper for given inlet heat energy or given total power output of the plant. For heat and power integration of process plant, heat engine should be appropriately placed against the process pinch to extract the most. The procedure described here are applicable for Carnot-like engine with linear heat transfer law. Generalization of this work may be applied for understanding real power plant better and design accordingly.

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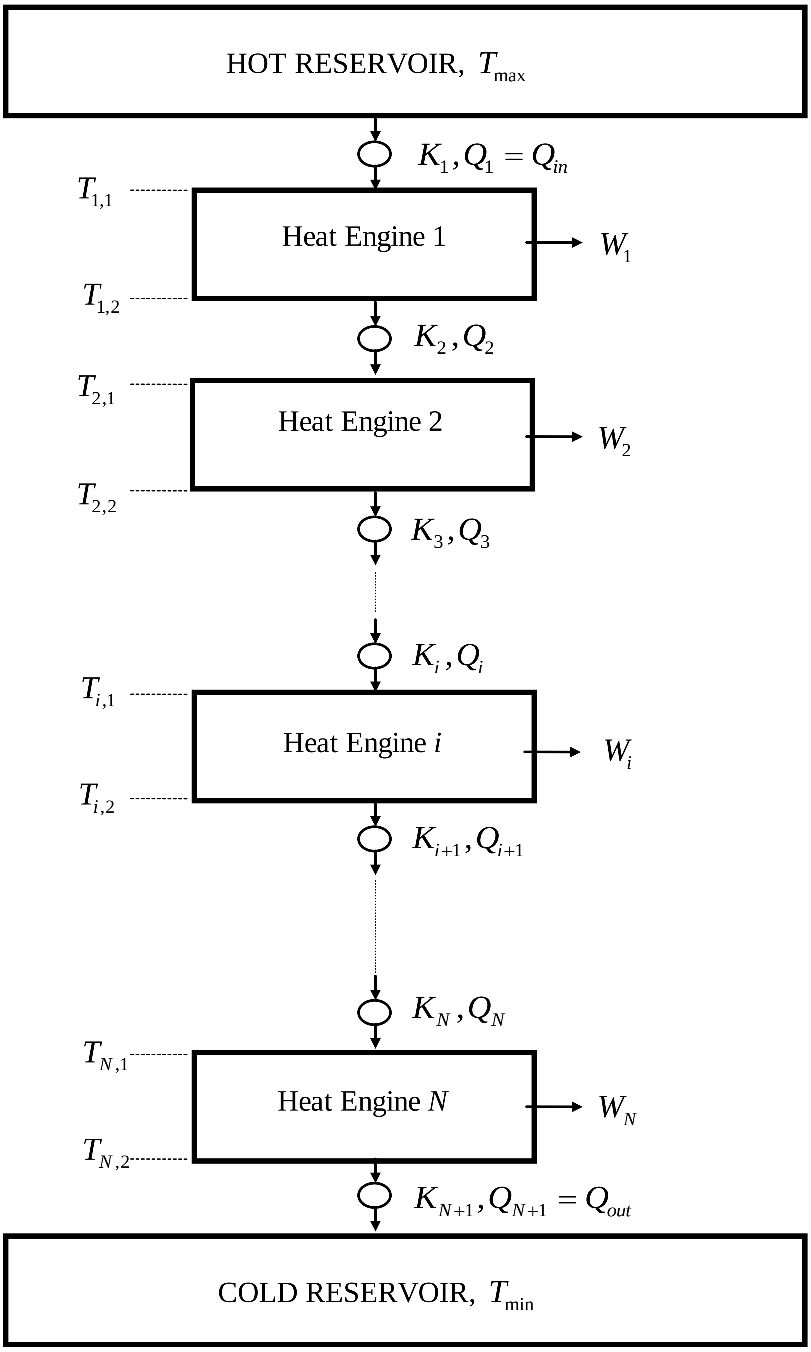

Figure 1. Schematic diagram of a multistage combined-cycle power plant comprising '*n*' irreversible Carnot-like heat engines