TEACHING THERMODYNAMICS THROUGH FALLACIES

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Abstract
Thermodynamics is an important subject in energy education. A thermodynamics course is particularly prone to difficulties, both in teaching as well as learning. Fallacies related to thermodynamics may pose fundamental challenges to young minds. It may be one of the interesting ways of teaching thermodynamics. In this paper a couple of fallacies related to thermodynamics are introduced. The first fallacy is related to thermodynamic optimisation and the second fallacy is about thermodynamic process. In the first fallacy, two different solutions for a thermodynamic optimization problem have been presented. In the second fallacy, it has been argued, that specific heat at constant pressure for any ideal gas is zero.

Keywords: Thermodynamics, pedagogy, fallacy, energy education

1. Introduction
One way of increasing the effectiveness of engineering education might be to expose young minds to different challenges. Paradoxes and fallacies related to a particular subject pose fundamental challenges to young minds. It has long been identified as one of the interesting ways of teaching any subject. Socrates' teaching method of perceiving new ideas was through fallacies and paradoxes. Greek philosophers also recognized this and created many paradoxes and fallacies. The great Greek geometer Euclid wrote an entire book on geometric fallacies which, unfortunately, has not survived (Gardner 1984).

Thermodynamics is an important subject in energy education. Thermodynamic analysis of a system allows the energy efficiency of the process to be quantified, regions with poor energy efficiency to be identified, and possible improvements to be defined. A thermodynamics course is particularly prone to difficulties, both in teaching as well as learning. It may be because in a thermodynamics course, it is not only necessary to master the physics of the problem and related governing equations, but also requires common sense to apply them appropriately (Müller, 2000). Levenspiel (1993) discussed about a fallacy in thermodynamics. In physical terms, he has argued, obviously fallaciously, that as you climb a mountain the air gets thicker, contrary to experience. Müller (2000) discussed about two conflicting solutions of a simple thermodynamic problem related to the uniform flow model.

It is important to distinguish fallacies from paradoxes. Paradox is a true observation or result though surprising while a fallacy is a false result obtained using reasoning that seems correct. Both paradoxes and fallacies are very interesting and instructive. In this paper we introduce a couple of fallacies related to thermodynamics. We expect that teaching thermodynamics through fallacies will not only make a student's training more accurate, but also it will be more exciting and fun for both students and teachers.

2. Fallacy-1: A Fallacy on Thermodynamic Optimization
Thermodynamic analysis has evolved into several subfields, viz., exergy analysis, lost-work analysis, entropy generation minimisation, finite time thermodynamics (FTT), etc. FTT originated with two independently published papers in 1957 (Novikov, 1957; Chambadal, 1957) and regained its popularity with another independent publication in 1975 (Curzon and Ahlborn, 1975). In recent years, FTT has attracted several criticisms. Most notable among them is the use of extremely simple models in highly idealised circumstances while ignoring important real-world issues (Moran, 1998). Rudimentary calculus and elementary algebraic manipulations are used to optimise such simple models. To validate the results obtained from such simple analysis, thermal efficiency of a large number of power plants is compared (Bejan, 1988). However, it should be kept in mind that such power stations were not designed to obtain the optimal thermal efficiency calculated by the FTT approach and hence cannot validate such results (Chen et al., 2001). In simplified engine model external irreversibilities only related to heat transfer are generally considered in FTT, ignoring important contributions of other internal and external irreversibilities. In fact, without assuming equal irreversible entropy changes in the

heat addition and rejection processes, an internally reversible Carnot like heat engine is not possible unless an appropriate heat leak analysis is performed (Sekulic, 1998). It is often argued that positive power cannot be produced from a reversible heat engine as reversible processes are defined only in limits of infinitely slow execution. Hence the term `finite' is used. However it's a misnomer and created confusion among scholars. Gyftopoulos (1999) gave couple of examples showing how infinitely slow processes can be more irreversible. Since internally reversible (also called endoreversible) heat engines consist of reversible processes, the concept of endoreversibility, itself, came under severe criticism (Sekulic, 1998). Bejan (1996) has pointed out that an important modelling limitation underlies the derivation of efficiency at maximum power by Curzon and Ahlborn (1975).

In the analysis of combined cycle power plants, contradicting results have been reported in literature. Wu et al. (1992) and Özkaynak (1995) have reported that the efficiency of a combined cycle heat engines while producing maximum power is more than that of a single cycle under similar conditions. Bandyopadhyay et al. (2001) have shown that a combined cycle heat engine produces less power at the maximum power condition, than a single heat engine. They have also shown that the efficiencies at maximum power are identical for both the cases. These results contradict each other and have resulted in a fallacy in the analysis and thermodynamic optimisation of combined cycle heat engines. To appreciate the fallacy, results related to a single cycle heat engine are briefly reviewed.

2.1 Analysis of a single heat engine

An internally reversible heat engine with irreversibilities assumed to be located at the heat exchangers, operates between a heat source at maximum temperature, T_{max} and a heat sink at minimum temperature, T_{min}. Schematic of such an endoreversible heat engine is shown in Figure 1a. It is bounded on either side by heat exchangers having overall thermal conductivities K_1 and K_2. The maximum and minimum temperatures of the working fluid are T_h and T_c, respectively. Rate of heat transferred through the heat exchangers and the entropy balance for the internally reversible heat engine are given as follows:

$$\dot{Q}_1 = K_1\left(T_{max} - T_h\right) \tag{1}$$

$$\dot{Q}_2 = K_2\left(T_c - T_{min}\right) \tag{2}$$

$$\frac{\dot{Q}_1}{T_h} = \frac{\dot{Q}_2}{T_c} \tag{3}$$

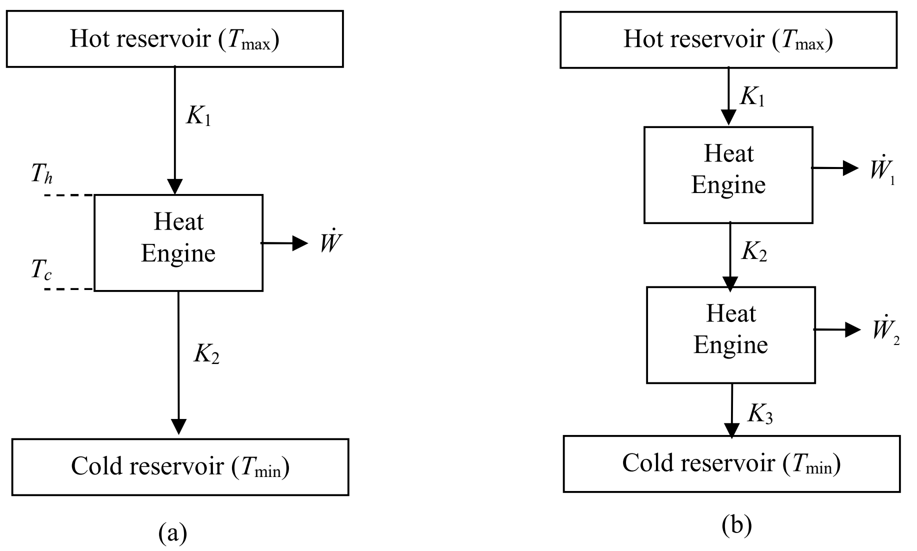

Figure 1. Schematic diagrams of (a) single endoreversible heat engine and (b) combined endoreversible heat engines.

Defining τ as T_c/T_h, equation (3) implies that $\tau = \dot{Q}_2 / \dot{Q}_1$. Combining equations (1) and (2) along with the definition of τ, T_h can be expressed as $\left(K_1 T_{max} + K_2 T_{min} / \tau\right)/\left(K_1 + K_2\right)$. Substituting the expressions for cycle temperatures, the power output of the engine can be written as

$$\dot{W} = \dot{Q}_1 - \dot{Q}_2 = \frac{(1-\tau)}{K_1^{-1} + K_2^{-1}} \left(T_{\max} - \frac{T_{\min}}{\tau} \right) \tag{4}$$

Maximisation of the power output with respect to τ results in $\tau_{opt} = \sqrt{T_{\min} / T_{\max}}$. The corresponding maximum power output and efficiency at maximum power of the engine are given by

$$\dot{W}_{\max} = \frac{\left(\sqrt{T_{\max}} - \sqrt{T_{\min}}\right)^2}{K_1^{-1} + K_2^{-1}} \tag{5}$$

$$\eta_{mP} = 1 - \sqrt{\frac{T_{\min}}{T_{\max}}} \tag{6}$$

Same procedure is now applied to the combined cycle consisting of two internally reversible heat engines in series.

2.2 First solution for combined cycle heat engines

A combined cycle with two internally reversible heat engines connected in series by a heat exchanger with overall thermal conductivity K_2, operates between a heat source at maximum temperature, $T_{\max}$ and a heat sink at minimum temperature, $T_{\min}$. The combined cycle is bounded on either side by heat exchangers having overall thermal conductivities K_1 and K_3. The maximum and minimum temperatures of the working fluid are T_{hi} and T_{ci}, respectively for both the engines. Schematic of such an endoreversible combined cycle is shown in Figure 1b. Following the procedure adopted in the last section, the power output of the combined cycle is given by:

$$\dot{W} = \dot{W}_1 + \dot{W}_2 = \frac{(1-\tau_1\tau_2)}{K_1^{-1} + K_2^{-1} + K_3^{-1}} \left(T_{\max} - \frac{T_{\min}}{\tau_1\tau_2} \right) \tag{7}$$

where τ_1 and τ_2 are defined as T_{c1}/T_{h1} and T_{c2}/T_{h2}, respectively. Maximum power is obtained at $(\tau_1\tau_2)_{opt} = \sqrt{T_{\max} / T_{\min}}$:

$$\dot{W}_{\max} = \frac{\left(\sqrt{T_{\max}} - \sqrt{T_{\min}}\right)^2}{K_1^{-1} + K_2^{-1} + K_3^{-1}} \tag{8}$$

The above equation shows that the power output of a continuous combined cycle engine is less than the power produced by a single engine working between the same source temperature $T_{\max}$ and sink temperature $T_{\min}$ (as $K_2 > 0$). The efficiency of the combined cycle heat engine corresponding to the maximum power condition remains same $\eta_{mP} = 1 - \sqrt{T_{\min} / T_{\max}}$. Entropy generation in the additional heat exchanger, incorporated in the combined cycle, results in reduction in power production. This may also be explained through Gouy-Stodola theorem. There is a proportional reduction of heat input to the combined cycle and hence, the efficiency at maximum power remains same. These results are consistent with those reported by Bandyopadhyay et al. (2001).

2.3 Second solution for combined cycle heat engines

Wu et al. (1992) and Özkaynak (1995) numerically analysed combined cycle power plants from finite time thermodynamics point of view. They reported that the efficiency of a combined cycle at maximum power is more than that of a single heat engine under similar conditions. In this section, the procedure adopted by these researchers is followed to reproduce those results analytically. These researchers started with the assumption that the maximum power produced by a combined cycle is the sum of the maximum power produced by the individual engines.

$$\dot{W}'_{\max} = \dot{W}'_{1\max} + \dot{W}'_{2\max} \tag{9}$$

Combining the phenomenological equations used in the last two sections, total power developed by the individual heat engines, in terms of temperature ratios τ_1 and τ_2 are (Babar et al., 2003)

$$\dot{W}_1 = \frac{(1-\tau_1)}{K_1^{-1}+K_2^{-1}+K_3^{-1}}\left(T_{\max} - \frac{T_{\min}}{\tau_1\tau_2}\right) \tag{10}$$

$$\dot{W}_2 = \frac{\tau_1(1-\tau_2)}{K_1^{-1}+K_2^{-1}+K_3^{-1}}\left(T_{\max} - \frac{T_{\min}}{\tau_1\tau_2}\right) \tag{11}$$

The optimum temperature ratio corresponding maximum power output of the first engine is calculated by equating the partial derivative of power, with respect to τ_1, with zero $\left(\partial\dot{W}_1/\partial\tau_1 = 0\right)$. Similarly, the optimum temperature ratio corresponding maximum power output of the second engine is calculated by equating the partial derivative of power, with respect to τ_2, with zero $\left(\partial\dot{W}_2/\partial\tau_2 = 0\right)$.

$$\left(\tau_1^2\tau_2\right)_{opt} = T_{\min}/T_{\max} \tag{12}$$

$$\left(\tau_1\tau_2^2\right)_{opt} = T_{\min}/T_{\max} \tag{13}$$

Combining equations (12) and (13), we get

$$(\tau_1)_{opt} = (\tau_2)_{opt} = \left(\frac{T_{\min}}{T_{\max}}\right)^{1/3} \tag{14}$$

The maximum power for the combined cycle power plant is

$$\dot{W}'_{\max} = \frac{1}{K_1^{-1}+K_2^{-1}+K_3^{-1}}\left(T_{\max}^{1/3}+T_{\min}^{1/3}\right)\left(T_{\max}^{1/3}-T_{\min}^{1/3}\right)^2 \tag{15}$$

The efficiency of the combined cycle at maximum power production is found to be

$$\eta'_{mP} = 1-\left(\frac{T_{\min}}{T_{\max}}\right)^{2/3} \tag{16}$$

Since ($T_{\min}/T_{\max}$) is less than unity, it can be shown that $\eta'_{mP} \geq \eta_{mP}$. Therefore, the efficiency of combined cycle at maximum power can be greater than that of a single engine. This confirms the results reported by Wu et al. (1992) and Özkaynak (1995).

2.4 Resolving the fallacy

Let us denote the minimum temperature ratio of the overall combined cycle heat engines, $T_{\min}/T_{\max}$ as γ. Now let us compare the power produced by the combined cycle as calculated by two different methods.

$$\frac{\dot{W}'_{\max}}{\dot{W}_{\max}} = \frac{\left(1+\gamma^{1/3}\right)\left(1-\gamma^{1/3}\right)^2}{\left(1-\gamma^{1/2}\right)^2} = \frac{1+\gamma-\gamma^{1/3}-\gamma^{2/3}}{1+\gamma-2\gamma^{1/2}} \tag{17}$$

Using the arithmetic mean-geometric mean inequality $\left[\left(\gamma^{1/3}+\gamma^{2/3}\right) > 2\gamma^{1/2}\right]$, it can be proved that

$$\dot{W}'_{\max} < \dot{W}_{\max} \tag{18}$$

This proves that the maximum power produced by the combined cycle in the later case is less than the maximum power that can be produced in the same cycle. The equation (18) shows that more power can be developed by the combined cycle when the cycle is optimised as a single unit rather than maximising individual cycles and then added together.

The model, $\dot{W}'_{\max} = \dot{W}'_{1\max} + \dot{W}'_{2\max}$ is incorrect and leads to the fallacious result. Correct approach should consider the complete power plant as a single unit, $\dot{W}_{\max} = \left(\dot{W}_1 + \dot{W}_2\right)_{\max}$. It may be noted that although the total power produced by the combined cycle is the linear sum of power produced by the individual engines, the maximum power produced by the combined cycle is *not* the linear sum of the maximum power produced by the individual engines. This also demonstrates the well-known fact that optimisation of individual subsystems need not lead to the optimisation of the overall system.

3. Fallacy-2: A Fallacy on Thermodynamic Process

Understanding of a process is very important in thermodynamics. Students may easily get confused between different processes and may draw wrong inference. A fallacy has been proposed in this section to improve the understanding of different thermodynamic processes. Unlike the first one where different solutions of a single problem have been reported, this fallacy is put to dramatise the effect. In the following fallacy it is argued that the specific heat at constant pressure for any ideal gas is zero.

3.1 The fallacy

An ideal gas of volume V_1 at pressure P_1 and temperature T_1 is compressed through an adiabatic isobaric process. After completion of the compression process, volume, pressure and temperature of the gas change to V_2, P_2 and T_2, respectively. As the process is isobaric (constant pressure), it may be written as

$$P_1 = P_2 \tag{19}$$

Change in internal energy of the gas may be expressed as

$$\Delta U = C_v \Delta T = C_v (T_2 - T_1) = C_v \left(\frac{P_2 V_2}{R} - \frac{P_1 V_1}{R} \right) = \frac{C_v P_1}{R} (V_2 - V_{1)}) \tag{20}$$

Net work done on the system may be calculated as

$$W_{1-2} = \int_1^2 P dV = P_1 (V_2 - V_1) \tag{21}$$

Applying the first law on the system, we get

$$Q_{1-2} = \Delta U + W_{1-2} \tag{22}$$

Since the process is adiabatic, there is no thermal interaction for the system. This implies that $Q_{1-2} = 0$. Combining this with the expression we obtained for change in internal energy (20) and work done (21), equation (22) may be simplified as follows.

$$0 = \frac{C_v P_1}{R} (V_2 - V_1) + P_1 (V_2 - V_1) \tag{23}$$

Since $V_1 \neq V_2$ and $P_1 \neq 0$, equation (23) implies that $C_v = -R$. For ideal gas it is known that $C_p = C_v + R$. Therefore, for an ideal gas going through an adiabatic isobaric compression, $C_p = C_v + R = -R + R = 0$.

3.1 Resolving the fallacy

In the previous demonstration, work done on the system has been calculated by integrating PdV. However, this is not true. This implies that for an adiabatic process, $\Delta U = -\int PdV$. This is true only if the process is reversible. Therefore, the actual work done on the system may be calculated to be $W_{1-2} = -\Delta U = C_v P_1 (V_1 - V_2) / R$.

4. Conclusions

Fallacies and paradoxes in thermodynamics always remind me of a popular remark, attributed to the great physicist Arnold Sommerfeld (Angrist and Helper, 1967). He is supposed to have said, “It’s a funny subject. The first time you go through it you don’t understand it all. The second time through you think you do except for one or two minor points. The third time you know you don’t understand it, but by then you are so used to it, it doesn’t bother you.” No doubt it is a subject difficult to understand and teach. Teaching thermodynamic through fallacies and paradoxes may improve understanding of thermodynamics.

In this paper two fallacies on thermodynamics are presented. The first fallacy is on thermodynamic optimisation. The fallacy is related to the maximum power production and efficiency at maximum power of a combined cycle power plant. Some researchers numerically solved and have reported that the efficiency of a combined cycle power plant is greater than that of a single heat engine at maximum power. Others have derived that the efficiencies of both single as well as combined cycle power plants are equal, at the maximum power condition. In this paper, this fallacy has been resolved analytically. The source of fallacy is the wrongly chosen model for optimisation. In the process, a pedagogical fact has also been reconfirmed that the optimisation of individual subsystems need not leads to the optimisation of the overall system. However, it may be noted that due to non-availability adequate optimization measures or tools, in practice the optimization of subsystems is nevertheless used for complex systems. The second fallacy is on thermodynamic process. It has been demonstrated fallaciously that specific heat at constant pressure for any ideal gas is identical to zero. The source of this fallacy is wrongly calculated value for work interaction. This fallacy tries to demonstrate the difference between an adiabatic process and a reversible adiabatic process.

Personally, I have limited scope of applying fallacies and paradoxes in teaching a thermodynamic course regularly. A few paradoxes and fallacies are demonstrated in an introductory course, Foundation of Energy Engineering, for post-graduate students perusing M.Tech. in Energy Systems Engineering at Indian Institute of Technology, Bombay. It has been observed that these paradoxes and fallacies lead to an increased interaction and many thought-provoking discussions beyond the scheduled lecture hours.

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