Modeling of Tube-Fin Shell and Tube Compact Heat Exchanger

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Abstract

Relations between effectiveness and number of heat transfer units (NTU) are derived for a novel tube-fin shell and tube compact heat exchanger. The derivation is carried out by dividing the overall exchanger into smaller units, represented by cross-flow exchanger with both fluids unmixed. Effects of different flow arrangements and number of baffles are also studied. These relations can be used as an effective tool by the designer to configure such a novel heat exchanger.

Keywords: heat exchanger, NTU-effectiveness relation, flow arrangements, Numerical solution.

Nomenclature

c	Ratio of rate of heat capacities.
$\dot{C}$	Rate of heat capacity (kW/K).
L	Length of one module (m).
NTU	Number of transfer units for one module.
q	Rate of heat transfer (kW).
t	Temperature (K).
U	Overall heat transfer coefficient for one module (kW/m^2 K).
W	Width of one module (m).
ε	Effectiveness.
Subscripts	
C	Cold.
E	Exit.
H	Hot.
i	Inlet.
m	Mean.
t	Tube fluid.
s	Shell fluid.

1. Introduction

Compact heat exchangers have high area density (of about 700 m^2/m^3) and provide higher flexibility in distributing heat transfer area as well as more structural stability [1]. A firm in Pune have developed a novel shell and tube heat exchanger with continuous fins on the outside of the tubes. It is essential for the designers to have effectiveness-number of heat transfer units (NTU) relations for such exchangers. Kays and London [2] have given detailed derivations of effectiveness-NTU relations for counter-flow, parallel-flow, multi-pass overall counter-flow and cross-flow heat exchangers. Underwood (1940), Fischer (1941), Bowman (1942) and Smith (1945) [as described in the ref. 3] have worked on F-factor correction for various shell and tube pass arrangements. Due to unavailability of any mathematical derivations, the manufacturing firm of this novel compact exchanger are currently obtaining the effectiveness-NTU relations through experiments. Therefore, a minor change in the design requires a fresh collection of data, which is costly and time consuming. In this paper, relations between effectiveness and NTU for such an exchanger with one tube and one shell pass are derived.

2. The Novel Heat Exchanger

This compact shell and tube heat exchanger has one shell side and one tube side pass (Fig. 1). The shell has evenly spaced baffles and there are continuous fins on the outside of the tubes. Overall heat transfer coefficient is assumed to be same throughout the exchanger. Each module-the space enclosed by any two baffles and the top and bottom tubes (Fig. 1)-represented by an approximate cross-flow exchanger with both-fluid-unmixed. The temperature profiles of the fluids are derived based on the work by Nusselt (1930) [as reported in ref. 3].

2.1. Governing Differential Equations

Assuming a constant overall heat transfer coefficient U, following relations can be written for the differential element shown in Fig. 2.

$$dq = U(t_t - t_s)dxdy \tag{1}$$

$$dq = -\frac{\dot{C}_t}{W}\frac{\partial t_t}{\partial x}dxdy \tag{2}$$

$$dq = -\frac{\dot{C}_s}{L}\frac{\partial t_s}{\partial y}dxdy \tag{3}$$

Following dimensionless variables are used: $\xi = x/L$, $\eta = y/W$, $\zeta_t = (t_t - t_{s,i,m})/(t_{t,i,m} - t_{s,i,m})$, $\zeta_s = (t_s - t_{s,i,m})/(t_{t,i,m} - t_{s,i,m})$, $a = US/\dot{C}_t$, $b = US/\dot{C}_s$ with $S = L.W$. The mean inlet temperatures of tube fluid and shell fluid are given as $t_{t,i,m}$ and $t_{s,i,m}$. Equating Eq. (1) with Eqs. (2) and (3), following differential equations are obtained.

$$\frac{\partial \zeta_t}{\partial \xi} + a\zeta_t = a\zeta_s \tag{4}$$

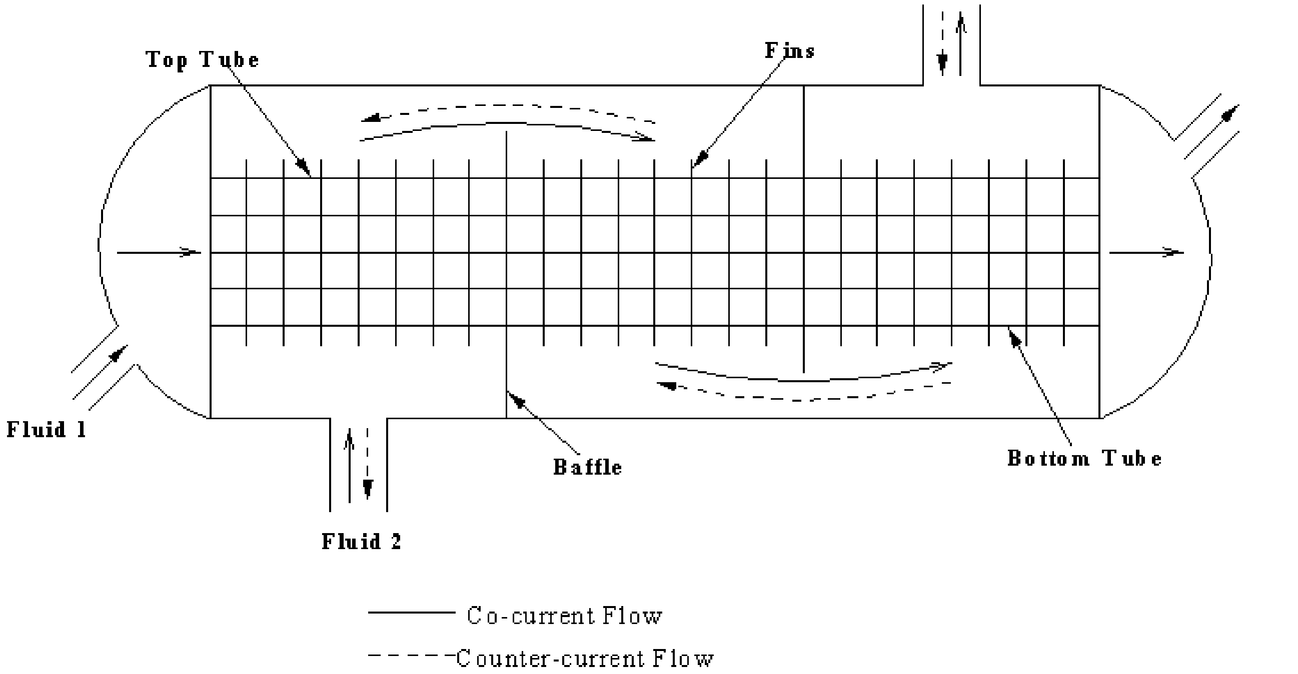

Fig. 1 Fin-tube compact shell and tube heat exchanger (one tube pass and one shell pass).

$$\frac{\partial \zeta_s}{\partial \eta} + b\zeta_s = b\zeta_t \tag{5}$$

In this heat exchanger the tube inlet temperature profile of a module is a function of η. On the other hand an assumption of through mixing of shell fluid over the baffles, will imply a uniform shell-side temperature profile for every module. These lead to the following boundary conditions:

$$\begin{aligned} \zeta_t &= \zeta_{t,i}(\eta) \qquad at \quad \xi = 0 \\ \zeta_s &= \zeta_{s,i}(\xi) \qquad at \quad \eta = 0 \end{aligned} \tag{6}$$

Applying the boundary conditions, solution of the governing differential equations can be written as:

$$\zeta_t = e^{-a\xi}\zeta_{t,i} + ae^{-a\xi}\int_0^{\xi}\zeta_s e^{a\xi} d\xi \tag{7}$$

$$\zeta_s = be^{-a\eta}\int_0^{\eta}\zeta_t e^{a\eta} d\eta \tag{8}$$

2.2. Temperature Profiles

To derive temperature profile for tube-side fluid, Eq. (8) will be substitute in Eq. (7).

$$\zeta_t = e^{-a\xi}\zeta_{t,i} + abe^{-(a\xi+b\eta)}\int_0^{\xi}\int_0^{\eta}\zeta_t e^{a\xi+b\eta} d\eta d\xi \tag{9}$$

Solution of this equation is give by

$$\zeta_{t,e} = \sum_{i=0}^{m-1}(a_i\psi^i)e^{\psi} + c_0 + \sum_{i=0}^{n-1}(b_i\psi^i)e^{-\psi} \tag{10}$$

The mean outlet temperature can be obtained as follows:

$$\zeta_{t,e,m} = \int_0^b\left(\sum_{i=0}^{n-1}a_i\psi^i e^{\psi} + c_0 + \sum_{i=0}^{m+n-1}b_i\psi^i e^{-\psi}\right)d\psi \tag{11}$$

Now from the definition of ζ_t and ζ_s, $\zeta_{t,e,m} = (t_{t,e,m} - t_{s,i,m})/(t_{t,i,m} - t_{s,i,m})$ and $q = \dot{C}_t(t_{t,i,m} - t_{t,e,m}) = \dot{C}_s(t_{s,e,m} - t_{s,i,m})$. Thus $t_{s,e,m}$ can be calculated. The coefficients a_i's, b_i's and c_0 are calculated through computer programs, developed during the study.

The outlet tube profile given by equation (10) can be converted to the inlet tube profile of the next module and the process can be repeated till the last module of the exchanger. In the case of counter-current flow iteration needs to be performed to get the final results.

The effectiveness-NTU relations for counter and co-current flows are presented and discussed below. The overall effectiveness, NTU and heat capacity are defined as follows: $\varepsilon = q/[\dot{C}_{\min}(t_{h,i} - t_{c,i})]$, $NTU = US/\dot{C}_{\min}$, and $c = \dot{C}_t/\dot{C}_s$.

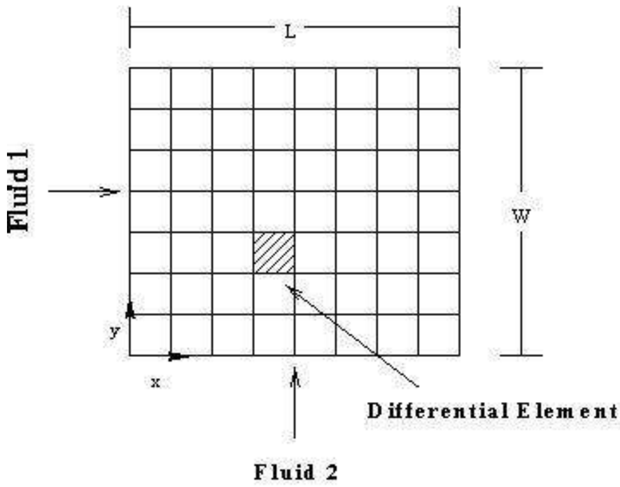

Fig. 2 Cross flow with both fluids unmixed (one module).

3. Results and Discussions

For an overall counter-current flow with a given NTU, with increasing modules (baffles) shell fluid inter-mixing increases, this brings the flow close to a pure counter-current resulting in an increase in the effectiveness (Fig. 3). Now larger the module NTU, greater the effect and therefore, the effectiveness increases with increasing NTU (Fig. 4). Note that, in an overall counter-current flow, effectiveness does not depends on which fluid passes through shell and which through tubes (Fig. 4).

For an over all co-current flow exchanger, when the module NTU of a heat exchanger is small, the mean outlet temperature of the cold fluid remains less then the mean outlet temperature of the hot fluid (i.e. temperature swapping does not takes place). An increase in overall NTU (or equivalently an increase in module NTU) leads to a higher heat exchange and therefore an increase in effectiveness. However at a particular module NTU (say NTU*) the mean outlet temperatures of the two fluids become equal and a further increase in module NTU results in temperature swapping. With module NTU higher than NTU*, effectiveness, either increases or decreases depending upon whether the number of modules are odd or even (Figs 5 and 6). This can be explained in the following manner: Consider a two module exchanger, in the first module there will be a temperature swapping so the mean outlet temperature of initially cooler fluid (say shell fluid) becomes more than the mean outlet temperature of hotter fluid. In the second module due to the second temperature swapping, the mean outlet temperature of the shell fluid (initially cooler) will decrease resulting a decrease in the overall effectiveness of the exchanger. In the case of three modules the third temperature swapping will increase the effectiveness and so on. With higher module NTUs this effect will become more prominent, therefore with increasing NTU, an exchanger with even number of modules shows a decreasing effectiveness and an opposite trend will be seen in the exchanger with odd number of modules. Note that, when the number of modules is odd, then for a given NTU higher effectiveness is obtained if the fluid with higher heat capacity flows through tubes (Fig. 5). However, it is other way round for even number of modules (Fig. 6).

4. Conclusion

Effectiveness-NTU relations for a novel compact shell and tube heat exchanger are derived mathematically for different flow directions. The variation in the effectiveness with the change in the number of baffles is also discussed.

When the overall NTU is small (approximately 1) the direction of the flow of fluids, the means of passage of fluids and the number of baffles have no effect on the effectiveness. Therefore the designer can choose these parameters as per the ease of design. However, if the overall NTU is large, then counter-current arrangement gives the maximum effectiveness. Although the means of passage of fluids has no effect on the effectiveness, but higher the number of baffles higher the effectiveness is. However, for C close to 1.0, with more than two baffles (three modules) there is not much increase in the effectiveness and also the maximum NTU (NTU_{max})–above which there is no substantial increase in effectiveness–is nearly 6.0. When C is near to extremes (say 10.0 or 0.1) NTU_{max} decreases from 6.0 to 4.0.

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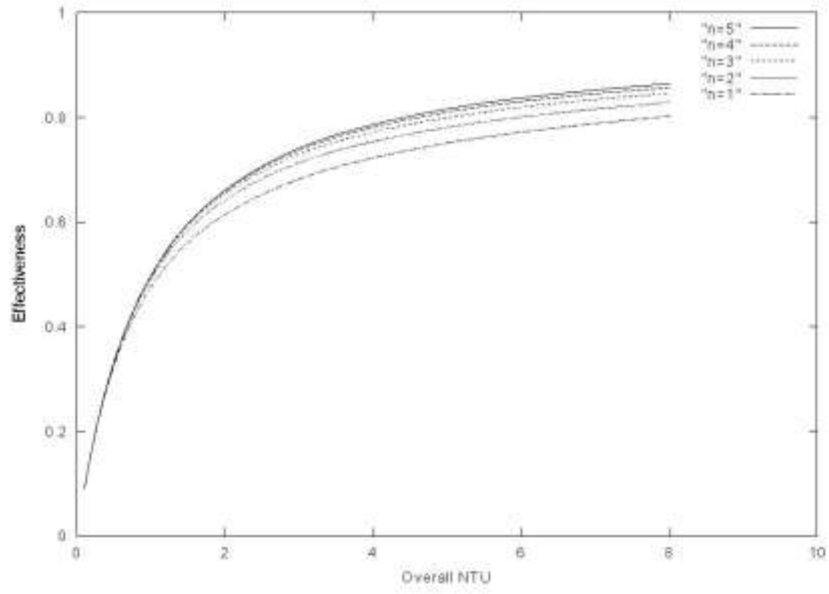

Fig. 3 Effectiveness vs. overall NTU for heat exchanger with different number of modules (counter-current flow) for C = 1.0.

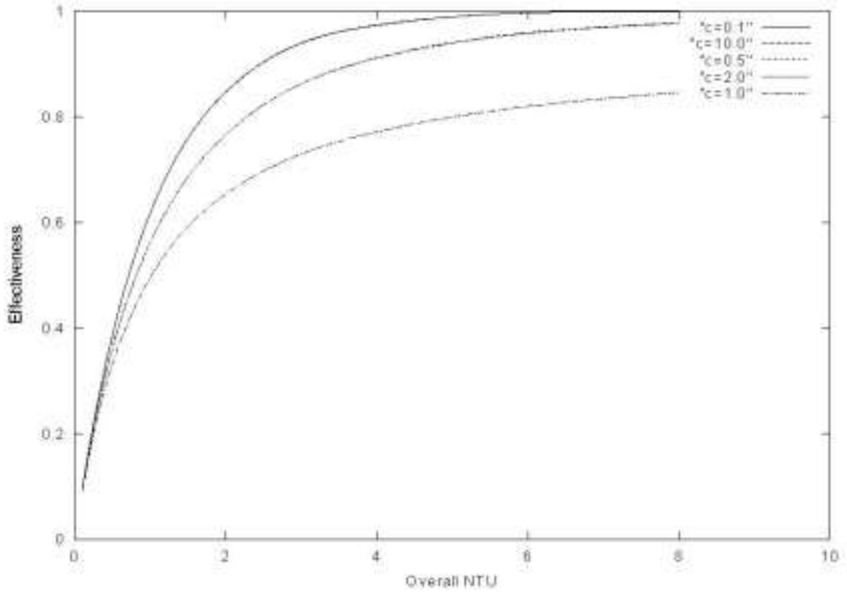

Fig. 4 Effectiveness vs. overall NTU for three module heat exchanger (counter-current flow).

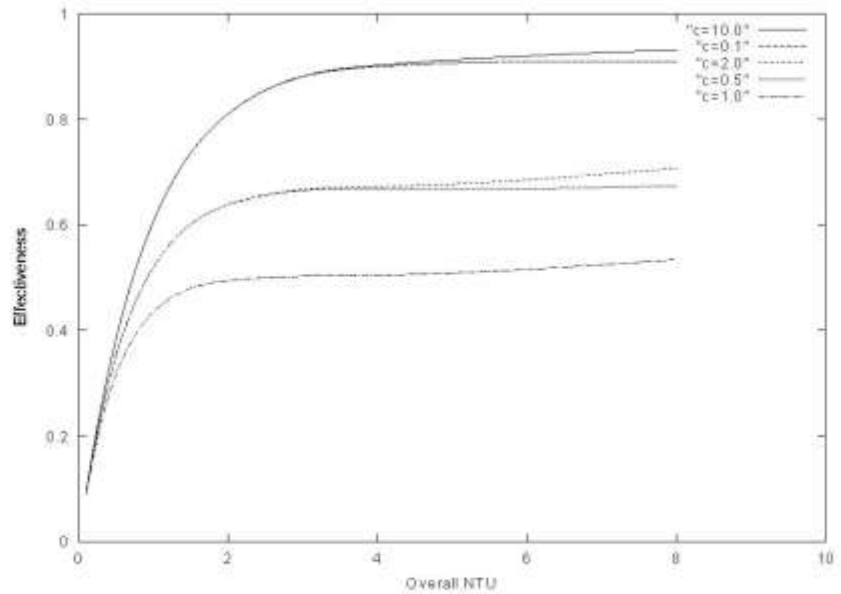

Fig. 5 Effectiveness vs. overall NTU for three module heat exchanger (co-current flow).

If the flow arrangement is to be co-current then the NTU_{max} is 1.8 to 2.5 (for C close to 1.0 to extremes of 0.1 or 10.0). Up to this NTU (i.e. 1.8 to 2.5), effectiveness does not depend upon the number of modules (or baffles) and the means of passage of the fluids. However if larger NTU is available at low cost then a slight improvement in the effectiveness can be achieved by passing higher heat capacity fluid through tubes and keeping the number of modules as one or three.

In the present derivation, two assumptions are made. First is assuming perfect mixing of shell fluid over the baffles. This assumption may not be true in cases where the fluid velocity over the baffles is low or the space above the baffle is large. In such cases an appropriate non-mixing must be taken into account. Second assumption is taking the heat transfer coefficient (U) constant over the whole exchanger. Again, changing fluid velocity by the introduction of baffles or switching of the shell and tube fluid will change the U. Thus a

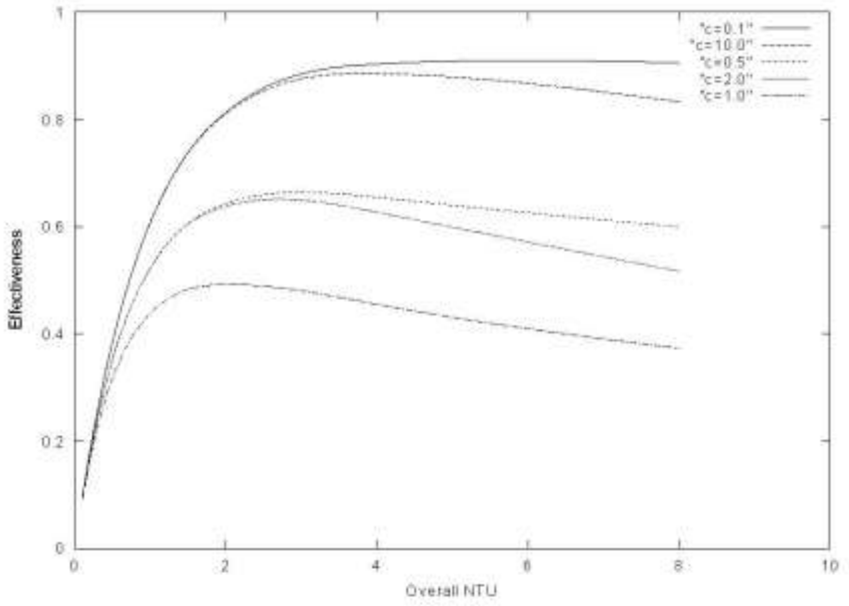

Fig. 6 Effectiveness vs. overall NTU for two module heat exchanger (co-current flow).

corrected heat transfer coefficient will give more practical results. Finally the current approach of dividing the exchanger into modules can also be extended to multiple shell pass and multiple tube pass exchanger.

5. References

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